

On Evolution Laws Taking Pure States to Mixed States in Quantum Field Theory

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Abstract

It has been argued that any evolution law taking pure states to mixed states in quantum field theory necessarily gives rise to violations of either causality or energy-momentum conservation, in such a way as to have unacceptable consequences for ordinary laboratory physics. We show here that this is not the case by giving a simple class of examples of Markovian evolution laws where rapid evolution from pure states to mixed states occurs for a wide class of states with appropriate properties at the “Planck scale”, suitable locality and causality properties hold for all states, and the deviations from ordinary dynamics (and, in particular, violations of energy-momentum conservation) are unobservably small for all states which one could expect to produce in a

laboratory. In addition, we argue (via consideration of other, non-Markovian models) that conservation of energy and momentum for all states is not fundamentally incompatible with causality in dynamical models in which pure states evolve to mixed states.

One of the most striking ramifications of the discovery that black holes should radiate as black bodies [1] is the implication that a black hole should completely evaporate within a finite time, and that, in this process, an initially pure quantum state should evolve to a mixed state (see, e.g., [2] for a review of arguments leading to this conclusion). This prediction of a “loss of quantum coherence” is derived in the semiclassical approximation by applying the ordinary dynamical evolution laws locally to the quantum field, so no violation of any of the principles of local quantum field theory occurs in this context. Indeed, in the semiclassical approximation, the loss of quantum coherence is directly attributable to the failure of the final time surface in the spacetime representing the evaporating black hole to be a Cauchy surface; exactly the same sort of phenomenon occurs when one considers the evolution of a free, massless field in Minkowski spacetime with the initial surface chosen as a ordinary hyperplane, but the final surface chosen as a hyperboloid.

Nevertheless, it seems natural to expect that when one goes beyond the semiclassical approximation, the possibility of loss of quantum coherence in black hole formation and evaporation should give rise to a significant (in principle) modification of the ordinary, local, dynamical evolution laws: For almost any initial quantum state, one would expect there to be a nonvanishing amplitude for black hole formation and evaporation to occur – at at least at a highly microscopic (e.g., Planckian) scale – thereby giving rise to a nonvanishing probability for evolution from pure states to mixed states [3], [4]. One would expect the deviations from the ordinary dynamical evolution laws to be negligibly small for all states normally accessible to laboratory experiments, but it would seem reasonable to expect large deviations to occur when, for example, the state of the quantum field is such that there is a substantial probability to produce black holes at the Planck scale.

It should be emphasized that any such modification of the ordinary dynamical laws which permits pure states to evolve to mixed states should fundamentally involve quantum gravity and, hence, a priori, there is no reason to expect it to be possible to adequately describe such effects by some “effective theory” in which spacetime structure is treated classically. A good analogy to bear in mind in this regard is the classical theory of electromagnetism with point

particles. Here, when radiation reaction effects are included, the theory is found either to violate causality or to have unacceptable consequences for laboratory physics (namely, runaway solutions). In this case, one does not normally view this nonexistence of an acceptable “effective classical theory” as an indication that quantum electrodynamics must be plagued by similar pathologies; rather, the usual interpretation of this situation is simply that a purely classical treatment of point particles is inadequate to describing radiation reaction phenomena. Similarly, a purely classical treatment of spacetime structure may be inadequate to describe phenomena in which pure states evolve to mixed states.

Indeed, it should be noted that black hole evaporation is an unusual process, in that during the formation of the black hole, the energy is transferred from the matter to the gravitational field of the black hole. The entropy of the matter, however, remains with the matter and is lost down the black hole, taking no energy with it. By contrast, for ordinary systems, the interactions with the environment which produce decoherence normally exchange both energy and entropy. Thus, the loss of coherence of a system normally is accompanied by energy non-conservation. What is needed in order to properly model what is believed to occur in the black hole case is to have an environment which can increase the entropy of a system, while at the same time exchanging energy with the system during only a limited interval of time, such that no net exchange of energy occurs. Thus, it should not be surprising that classical spacetime models which have not been carefully designed to do this (see the Appendix) will face difficulties with energy conservation. However, this does not imply that similar difficulties with energy conservation need occur in processes involving black holes.

In any case, it is of interest to know if – in the context of theories where spacetime structure is treated classically – there exists any difficulty, in principle, in finding mathematically well defined “effective dynamical evolution laws” such that a suitable class of pure states can rapidly evolve to mixed states, but no inconsistencies with known laboratory physics occur. If a fundamental difficulty is present, then – despite the comments in the previous two paragraphs – this could conceivably indicate the presence of a similar difficulty in quan-

tum gravity. This issue was addressed by Banks, Susskind, and Peskin [5], who, following an earlier analysis of Ellis et al [3], argued that a serious difficulty of principle does exist. They concluded that any dynamical evolution law which takes pure states to mixed states (with appropriately large probability for suitable states) must give rise to unacceptably large violations of causality or energy-momentum conservation at the scales of laboratory physics (however see [6]). Although these authors did not claim to provide a complete proof of this conclusion, their arguments have gained widespread acceptance and appear to underlie many efforts to modify the picture of black hole formation and evaporation provided by the semiclassical approximation, so that an initially pure quantum state will remain pure in that process.

In this paper, we shall re-examine the arguments of [5] and draw the opposite conclusion: We will consider what is, in essence, simply a subclass of the models considered in [5] with good causal properties, and will show that they can be adjusted to yield an arbitrarily rapid loss of quantum coherence for states with suitable properties at, say, the Planck scale, but produce a negligible deviation from ordinary dynamical evolution for states which can be produced in laboratories. Thus, although in these models violations of energy and momentum conservation presumably would occur (as argued in [5]) and Lorentz invariance presumably also would fail [7], we shall show that (contrary to the claims made in [5]) there is no difficulty confining such pathologies to the “Planckian states”, which are not accessible to ordinary laboratory physics.

Although we believe that the models considered in the body of the paper below would predict violations of energy-momentum conservation if one did scattering experiments with particles of Planck scale energy, it should be emphasized that these models (as well as the somewhat more general models considered in [5]) encompass only “Markovian” models, where the equation of motion governing the time evolution of states is local in time; more precisely, the time evolution map has the structure of a dynamical semigroup (see, e.g., [8]). Since a black hole should have a long timescale “memory” (stored in its external gravitational field) of the amount of energy that went into it, one would not expect an effective evolution

law modeling the process of black hole formation and evaporation to be Markovian in nature. In the Appendix, we shall consider some alternative, non-Markovian models. Although these models are not satisfactory as models of the black hole formation and evaporation process, they serve the purpose of showing that causality and energy-momentum conservation are not fundamentally in conflict for evolution laws taking pure states to mixed states, i.e, one can construct classical spacetime models in which pure states evolve to mixed states and energy-momentum conservation and causality hold on all scales, not merely on the scales of laboratory physics.

To construct our Markovian models displaying rapid loss of quantum coherence for “Planckian states”, but negligible deviation from ordinary dynamics for “laboratory states”, we start with an ordinary, causal, unitary, local quantum field theory in Minkowski spacetime – such as a free Klein-Gordon field – with dynamics determined by a Hamiltonian H . We then consider the following class of modified dynamical laws,

$$\dot{\rho} = -i[H, \rho] - \sum_i \lambda_i (Q_i \rho + \rho Q_i - 2Q_i \rho Q_i) \quad (1)$$

where each λ_i is positive and where each Q_i is an orthogonal projection operator (i.e., $Q_i^\dagger = Q_i$ and $Q_i^2 = Q_i$). This modified dynamical law is simply a specialization of the general form of a Markovian evolution law given in eq. (4.3) of [8] to the case where the operators V_j appearing in that equation are projection operators; it also corresponds to eq.(9) of [5], specialized to the case where their matrix $h_{\alpha\beta}$ is diagonal and their Q 's are projection operators. We do not impose any additional conditions upon the different Q_i 's at this stage, although we shall impose an additional locality restriction below to assure suitable causality properties of the theory; in particular, note that the Q_i 's are not assumed to commute.

Taking the trace of eq.(1), we immediately obtain $\text{tr} \dot{\rho} = 0$. Furthermore, the arguments of [5] show that eq.(1) preserves the positivity of ρ , and that it decreases the purity of states in the sense that $d/dt[\text{tr}(-\rho \ln \rho)] \geq 0$. Thus, eq.(1) evolves pure states into mixed states in such a way as to conserve probability and keep all probabilities positive.

Since eq.(1) is linear in ρ , it defines a linear time evolution operator $\$(t_0, t)$ on density matrices, so that, in index notation, we have

$$\rho^A{}_B(t) = \$^A{}_{BC}{}^D(t_0, t) \rho^C{}_D(t_0) \quad (2)$$

The Heisenberg representation version of this dynamics (where the states are viewed as fixed in time) is obtained by evolving each observable, A , by a suitable transpose of $\$$, namely

$$A^A{}_B(t) = \$^C{}_{DB}{}^A(t_0, t) A^D{}_C(t_0) \quad (3)$$

This corresponds to the following Heisenberg equation of motion for A ,

$$\begin{aligned} \dot{A} &= +i[H, A] - \sum_i \lambda_i (Q_i A + A Q_i - 2Q_i A Q_i) \\ &= +i[H, A] + \sum_i \lambda_i [Q_i, [A, Q_i]] \end{aligned} \quad (4)$$

Equation (4) will be useful for our analysis below of the locality properties of the model [9].

Some insight into the nature of the dynamics defined by eq.(1) can be gained by considering the special case where $H = 0$ and only one of the Q'_i s (denoted Q) is present. In this case, we decompose ρ into a 2×2 block matrix with respect to the subspace defined by Q and its orthogonal complement. It then is easily seen that the evolution law eq.(1) leaves the diagonal blocks of ρ unchanged but causes the off-diagonal blocks to exponentially decay away. Thus, in this case, eq.(1) corresponds simply to a decoherence between the subspace associated with Q and its orthogonal complement. Of course, a much richer dynamics can occur when $H \neq 0$ and when many noncommuting Q'_i s are present.

We now shall further specialize eq.(1) to give the model suitable locality properties. This will allow us to keep violations of causality under control (even for “exotic” states where rapid evolution from pure to mixed states occurs) and enable us to ensure that possible exotic phenomena occurring in distant regions of the universe will not affect observations in our laboratories. Let \mathcal{R} be any region of space and let R be any local field observable for this region. (A good example of such an R of possible relevance to black hole formation issues would be obtained by integrating the 4-momentum density of the field over \mathcal{R} and

then squaring it.) Let Q be a projection operator onto a subspace spanned by eigenvectors of R ; for definiteness, we choose a real number, α , and take Q to be the projection operator onto the subspace of eigenvectors of R with eigenvalues greater than α . Now, let \mathcal{T} be any spatial region disjoint from \mathcal{R} , and let T be any local field observable for \mathcal{T} . Then T commutes with R and, hence, T commutes with all of the projection operators occurring in the spectral resolution of R . Hence, in particular, $[Q, T] = 0$. Since the unmodified, unitary quantum field theory has causal propagation, the Heisenberg representative of T in the unmodified theory will commute with Q until such time as a light signal from \mathcal{T} can reach \mathcal{R} . Now, consider the modified, non-unitary Schrodinger dynamics defined by

$$\dot{\rho} = -i[H, \rho] - \lambda(Q\rho + \rho Q - 2Q\rho Q) \quad (5)$$

which corresponds to the Heisenberg dynamics

$$\dot{T} = +i[H, T] + \lambda[Q, [T, Q]] \quad (6)$$

for T . By inspection, the solution to the Heisenberg equation of motion for the unmodified theory solves eq.(6) until a light signal from \mathcal{T} can reach \mathcal{R} . This implies that starting from any initial (globally defined) state at $t = 0$, an observer in the theory defined by eq.(5) who makes local measurements in the region \mathcal{T} will not be able to detect any difference from ordinary dynamics until effects from region \mathcal{R} can causally propagate to him. In other words, the dynamics defined by eq.(5) is entirely “ordinary” (and, in particular, local and causal) outside of the region \mathcal{R} . If we now choose \mathcal{R} to be so small (e.g., Planck dimensions) that it is inaccessible to laboratory measurements, no departures whatsoever from causality and locality will be detectable.

We now restrict our model so that each projection operator, Q_i , appearing in eq.(1) is a projection operator for a local observable R_i associated with an inaccessibly small region of space \mathcal{R}_i . (We also may allow “large” regions, \mathcal{R}_i , provided that we then choose the corresponding λ_i to be sufficiently small.) If the different regions, \mathcal{R}_i , are allowed to overlap, observable violations of causality could still occur in the theory (due to “chain reactions”).

However, it is clear that even these potential violations can be kept under good control by imposing only mild restrictions on both the overlap of the regions and the values of the λ'_i s. Thus, we claim that there is no difficulty in adjusting the model so that the resulting dynamics will be observably local and causal for essentially all states, including those for which the rate of loss of quantum coherence is large.

The restriction we have placed upon the Q'_i s, corresponds, in essence, to a discretized version of eq.(20) of [5], with the support of their spatial smearing function $h(x - y)$ chosen to be small. This restriction on the dynamics also was imposed in [5] to ensure good causal propagation properties of the model. Hence, we have not, thus far, diverged in any substantial way from the analysis of [5]. The authors of [5] then argue further that violations of energy-momentum conservation must occur. We see no reason to question this conclusion for the above model, although in the Appendix we will show that other, non-Markovian models need not suffer from this problem. The authors of [5] then claim that these violations will be so large as to have a drastic effect upon ordinary laboratory physics. No general arguments are given in [5] to support this assertion, but an example is worked out which illustrates this phenomenon. However, we now shall show that there is no difficulty in tailoring the dynamics defined by eq.(1) (with our above restriction on the Q'_i s) so that all violations of energy-momentum conservation (as well as any other deviations from ordinary dynamics) would not be detectable in laboratory experiments.

To do so, we identify a subspace, \mathcal{H}_L , of the Hilbert space of states corresponding to those states accessible to laboratory physics, and we also choose a collection of “inaccessibly small” regions of space \mathcal{R}_i along with corresponding observables, R_i . The details of these choices are not of great importance, provided only that (i) the subspace \mathcal{H}_L is mapped into itself by the ordinary, unitary dynamics defined by H and (ii) the regions \mathcal{R}_i are chosen sufficiently small that the states in \mathcal{H}_L restricted to any \mathcal{R}_i do not differ grossly from some representative state, say, the vacuum state $|0\rangle$. A concrete example of such choices in Klein-Gordon theory would be to take \mathcal{H}_L to be the subspace of states spanned by particles whose mode functions contain no frequencies higher than ω_0 (where ω_0 is chosen to be much

higher than is achievable in laboratories) and to take each \mathcal{R}_i to have size much smaller than ω_0^{-1} . We now choose (some of) the λ_i 's to be as large as we like, thus ensuring that arbitrarily rapid loss of quantum coherence occurs for some states. Finally, we choose each Q_i so that each $\epsilon_i \equiv \lambda_i ||Q_i|0 > ||^2$ is as small as we wish. This latter condition can easily be achieved by choosing α_i sufficiently large (so that Q_i is a projection onto a subspace of extremely large eigenvalues of R_i). It is worth noting that since for typical local observables $||Q_i|0 > ||^2$ will fall-off very rapidly with α_i – for example, $||Q_i|0 > ||^2$ will be essentially Gaussian in α_i if R_i is the field operator smeared over \mathcal{R}_i – it normally will not be necessary to choose α_i to be significantly larger than the scales associated with \mathcal{R}_i in order to obtain the desired smallness of ϵ_i . Since the “laboratory states” do not differ greatly from $|0 >$ with respect to the observables R_i , this will ensure that $\lambda_i tr(\rho Q_i)$ is similarly small for any density matrix ρ constructed from the “laboratory states”. It then follows immediately that the probability that eq.(1) will result in an observable difference from the unmodified, unitary dynamics for “laboratory states” will be negligible.

It is useful to compare the analysis of the above paragraph to the illustrative model given in section 5 of [5]. In essence, the model of [5] differs from the above models only in that our projection operator Q_i is replaced by the squared field operator at a point (made finite by the imposition of a momentum cutoff at the Planck scale). However, this squared field operator applied to the vacuum state yields a state with large norm, so one must choose the parameter a of their model (corresponding to our λ_i) to be exceedingly small in order to avoid affecting ordinary laboratory dynamics. With a chosen to be this small, non-unitary dynamics occurs only for states with exotic properties at energy scales far in excess of the Planck scale. However, this difficulty of the model could be avoided by replacing the squared field operator by a suitable function of the field operator, where the function is chosen so that it is (significantly) non-zero only for values of the field which are sufficiently large that vacuum fluctuations to that value of the field are highly improbable. A step function (yielding a projection operator) is ideal for this purpose.

In summary, we have shown that even the simple class of Markovian models considered

here is sufficient to encompass dynamics which differs imperceptibly from ordinary dynamics for all states that have properties similar to the vacuum state at extremely small scales, but is highly non-unitary for states that differ greatly from the vacuum state on these scales. In essence, our analysis differs from [5] only in that we have developed their basic model sufficiently that one can explicitly see how to independently control the values of the quantities λ_i (which determine the maximum rate of loss of quantum coherence) and ϵ_i (which determine the probabilities for observing a loss of quantum coherence for laboratory states). We conclude that there appears to be no difficulty of principle in constructing theories which capture the essential features that might be expected if black hole formation and evaporation at a highly microscopic scale occurs in the manner suggested by the semiclassical picture.

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Appendix

The models considered above are Markovian in nature. Such models arise naturally as an effective dynamics of a system coupled to another large system (i.e., “heat bath”) in the limit where the relaxation time of the heat bath goes to zero. The Markovian character of an effective evolution law makes it difficult to lose coherence while conserving energy exactly. However, one would not expect the effective dynamics corresponding to the process of black hole formation and evaporation to be Markovian, since the black hole should “remember” (via its external gravitational field) the amount of energy which was dumped into it, and it should be able to return this energy via particle creation at very late times. Indeed, this is especially true if correlations *are* restored during the late stages of black hole evaporation, since this would require an exceedingly long “relaxation time” of the black hole system. In this Appendix, we will analyze two simple non-Markovian models for the loss of coherence, which provide good illustrations of our claim that there need not be any conflict between loss of coherence, causality, and energy-momentum conservation. The basic idea of both

models is to have a “hidden system” interacting with the given system. This hidden system will have no energy of its own and therefore will not be available as either a net source or a sink of energy. However, the state of that hidden system will affect the behaviour of the system of interest, in such a way as to produce a loss of quantum coherence in the system of interest.

For the reasons indicated at the end of this Appendix, the models we treat here are not satisfactory as models of the black hole evaporation process. However, it should be noted that the loss of coherence without energy loss has also been studied in realistic condensed matter systems [10]. There the hidden sector which causes the decoherence is taken to be the nuclear spins of the atoms making up the system. Because of the weak interactions of the nuclear spins with each other, they comprise a zero energy sink which can however correlate both with the state and the history of the system of interest, leading to loss of coherence without loss of energy. This phenomenon plays a crucial role in the physics of such systems.

Our first model involves the interaction of a quantum field with a simple harmonic oscillator, where the frequency of the oscillator depends upon the state of a spin system. (Both the oscillator and the spin system comprise the “hidden system” in this model.) The resonant scattering of the field will thus depend on the state of the spin system, leading to decoherence for that scattering process. By placing the harmonic oscillator at some fixed point in space, the interaction and loss of coherence will clearly be local in space, but the complete system will fail to be translation invariant. Consequently, this model will be one in which the field loses coherence without any violation of causality or energy conservation, but where momentum conservation fails.

Our second model is simply ordinary $\lambda\phi^4$ field theory except that we now treat λ as a random variable (which we may view as representing hidden degrees of freedom), with probability distribution $R(\lambda)$. In this case, loss of quantum coherence will occur in scattering in such a way as to be entirely causal and satisfy conservation of energy and momentum.

For our first model we consider a scalar field in one spatial dimension which interacts

with a simple harmonic oscillator located at the origin. Radiation can excite the oscillator, which then subsequently decays, reemitting its radiation. Such a model clearly conserves energy, since all of the energy absorbed by the oscillator is reemitted. However, in this bare form, there is no loss of coherence, since the field eventually regains all of the coherence which is lost in the intermediate states when one traces over the oscillator, as has been analysed by Anglin, Laflamme, Zurek, and Pas [11]. Indeed, this model is a simplified form of a model in which a lump of matter is heated by radiation, and then cools down again. To create a genuine loss of coherence, we couple the oscillator to an internal system, which for simplicity, we take to be a spin system with total spin s . The spin system is assumed to have no free Hamiltonian, but gains an energy only if the oscillator is excited. The total Hamiltonian is taken to be

$$H = \frac{1}{2} \int \left((\pi(t, x) - h(x)q)^2 + (\partial_x \phi(t, x))^2 \right) dx + \frac{\omega}{2} (p^2 + q^2 - \frac{1}{2}) (1 + \alpha(S_z + s)) + F(S_z) \quad (7)$$

where the function $h(x)$ is sharply peaked around $x = 0$, and will be treated as being proportional to a δ function. S_z is the z component of the spin operator for the spin s system. Note that H commutes with S_z , so we may consider the spectrum of H separately in each sector of eigenstates of S_z . We choose the operator $F(S_z)$ so that the energies of the lowest lying state in each of these sectors are identical.

Now, suppose that the spin system is in an eigenstate of the operator S_z with eigenvalue m . Then the energy levels of the oscillator are of magnitude $\omega((m + s)\alpha)$. That oscillator will therefore absorb and reemit radiation with frequencies around $\omega((m + s)\alpha)$. Thus, the scattered radiation from that oscillator will depend on the state of the hidden spin. This implies that if this spin system is not initially in an eigenstate of S_z , correlations will develop between the spin system and the quantum field. If we trace over the states of this spin, the field will, in general evolve into a mixed state.

To see this explicitly, we note that the solution to the equations of motion for the above model (taking $h(x) = h\delta(x)$) is given by

$$\phi(t, x) = \phi_{in}(t, x) + \frac{1}{2} h q(t - |x'|) dx' \quad (8)$$

$$q(t, x) = \int^t e^{-\gamma(t-t')} \frac{1}{\Omega_m} \sin(\Omega_m(t-t')) h \dot{\phi}_{in}(t', 0) dt' \quad (9)$$

where ϕ_{in} is a free field operator, $\gamma = h^2/4$, and $\Omega_m^2 = (\omega(\alpha(m+s))^2 - \gamma^2$. Thus we have

$$\phi(t, x) = \phi_{in}(t, x) + \frac{\gamma}{\Omega_m} \int^t e^{-\gamma(t-t')} \sin(\Omega_m(t-t')) \dot{\phi}_{in}(t', 0) dt' \quad (10)$$

Examining the scattering states, we find that the outgoing annihilation operators of the field are related to the ingoing annihilation operators by

$$a_{k,out} = a_{k,in} - i\sigma_{k,m}(a_{k,in} + a_{-k,in}) \quad (11)$$

or reversing the flow,

$$a_{k,in} = a_{k,out} + i\sigma_{k,m}(a_{k,out} + a_{-k,out}) \quad (12)$$

where

$$\sigma_{k,m} = i \frac{2\gamma|k|}{-|k|^2 + 2i\gamma|k| + \Omega_m^2} \quad (13)$$

Thus, the outgoing state $|\psi_m\rangle$ when the field initially is in the state $\int \beta(k) a_{k,in}^\dagger |0\rangle$, the harmonic oscillator initially is in its ground state, and the spin system initially is in the state $|m\rangle$, is given by

$$|\psi_m\rangle = \int \beta_k (a_{k,out}^\dagger + \sigma_{k,m}^* (a_{k,out}^\dagger + a_{-k,out}^\dagger)) dk |0\rangle \quad (14)$$

Clearly, the complete model is causal (the field propagates away from the oscillator at the speed of light), energy conserving, and unitary. However, when we start with the spin system in the state $|\kappa\rangle = \sum_m \kappa_m |m\rangle$ the reduced density matrix of the field at very late times will be given by

$$\rho = \sum_m |\kappa_m|^2 |\psi_m\rangle \langle \psi_m| \quad (15)$$

To see at least an effect of the loss of coherence we can look at

$$Tr(\rho^2) = \sum_m \sum_{m'} |\kappa_m \kappa_{m'}|^2 |< \psi_m | \psi_{m'} >|^2 \quad (16)$$

the square of the density matrix, whose deviation from unity gives a measure of the coherence lost in the scattering process. We have

$$< \psi_m | \psi_{m'} > \quad (17)$$

$$= < 0 | \int \beta^*(k) (a_{out,k} + \sigma_{|k|,m}^* (a_{out,k} + a_{out,-k})) dk \quad (18)$$

$$\times \int \beta(k') (a_{out,k}^\dagger + \sigma_{|k|,m'} (a_{out,k'}^\dagger + a_{out,-k'}^\dagger)) dk' | 0 > \quad (19)$$

$$= \int |\beta(k)|^2 \left((1 + \sigma_{|k|,m}^* + \sigma_{|k|,m'} + \sigma_{|k|,m}^* \sigma_{|k|,m'}) \right. \quad (20)$$

$$\left. + \sigma_{|k|,m}^* \sigma_{|k|,m'} \right) dk \quad (21)$$

where we have assumed that $\beta(k)$ is non-zero only for $k > 0$. Thus we finally have

$$Tr(\rho^2) = 1 + \sum_m 2|\kappa_m|^2 \int |\beta(k)|^2 Re(\sigma_{|k|,m}) dk \quad (22)$$

$$+ 2 \sum_{m,m'} |\kappa_m|^2 |\kappa_{m'}|^2 \int |\beta(k)|^2 \sigma_{|k|,m}^* \sigma_{|k|,m'} dk \quad (23)$$

Since $< \psi_m | \psi_m > = 1$, we also have that

$$2 \int |\beta(k)|^2 Re(\sigma_{|k|,m}) dk = - \int |\beta(k)|^2 \sigma_{|k|,m}^* \sigma_{|k|,m} dk \quad (24)$$

so that

$$Tr(\rho^2) = 1 - 2 \sum_{mm'} |\kappa_m|^2 |\kappa_{m'}|^2 \int |\beta(k)|^2 \left(|\sigma_{|k|,m}|^2 - \sigma_{|k|,m} \sigma_{|k|,m'}^* \right) dk \quad (25)$$

Thus, if the various $\sigma_{k,m}$ do not overlap, this becomes

$$Tr(\rho^2) = 1 - 2 \sum_m (|\kappa_m|^2 - |\kappa_m|^4) \int |\beta(k)|^2 \sigma_{|k|,m}^* \sigma_{|k|,m} dk \quad (26)$$

If they do overlap, the sum will be increased from this expression. (If they are independent of m , $Tr(\rho^2)$ is just unity, as would be expected.) We can thus have a large reduction in coherence if the various κ_m are sufficiently small.

For our second model, we start with ordinary $\lambda\phi^4$ field theory, and we let $U_\lambda(t_0, t)$ denote the unitary time evolution operator of this theory. We now define a new dynamical evolution law by choosing a time t_0 (which we may take to be $t_0 = -\infty$) and setting

$$\rho^A_B(t) = \$^A_{BC}{}^D(t_0, t) \rho^C_D(t_0) \quad (27)$$

where

$$\$^A_{BC}{}^D(t_0, t) = \int R(\lambda) U^A_{\lambda C}(t_0, t) U^{\dagger D}_{\lambda B}(t_0, t) d\lambda \quad (28)$$

and $R(\lambda)$ is an arbitrarily chosen probability distribution, (so $R(\lambda)$ is non-negative and $\int R(\lambda) d\lambda = 1$). Thus, the new dynamical evolution law corresponds to $\lambda\phi^4$ theory with λ being a random variable. (Note that the dynamical evolution law for the quantum field obtained in the above oscillator model also is of the general form (28), and the non-Markovian character of both models is manifested by the fact that $$(t_0, t) \neq $(t_1, t)$$(t_0, t_1)$.) It is easily seen that this dynamical evolution law takes initial pure states to mixed states. Indeed, for a pure initial state $|\Psi_0\rangle$ at time t_0 , the state at time t is given by$

$$\rho(t) = \int R(\lambda) |\Psi(t; \lambda)\rangle \langle \Psi(t; \lambda)| d\lambda \quad (29)$$

where $|\Psi(t; \lambda)\rangle = U_\lambda(t_0, t) |\Psi_0\rangle$. We have

$$Tr \rho^2 = \int \int R(\lambda) R(\lambda') |\langle \Psi(t; \lambda) | \Psi(t; \lambda') \rangle|^2 d\lambda d\lambda' \quad (30)$$

which is not equal to unity unless the states $|\Psi(t; \lambda)\rangle$ are independent of λ .

Since $\lambda\phi^4$ theory is causal for each value of λ , it is manifest that the new dynamics is causal, i.e., no observers can use the ϕ -field to send signals faster than light. Furthermore, since energy and momentum are conserved for each λ and these observables may be replaced by their free field ($\lambda = 0$) values after the interactions have occurred, we see that energy and momentum (as well as angular momentum) are exactly conserved in all scattering experiments. The model also is Lorentz covariant. Thus, this model explicitly demonstrates that loss of quantum coherence is not incompatible with all of the above properties.

It should be noted that this model (as well as the previous oscillator model) would have some unsatisfactory features as a model of the black hole evaporation process. Specifically, one would expect the results of an experiment involving black hole formation and evaporation to be uncorrelated with the results of similar experiments performed at other times or places.

However, since λ is constant over spacetime, such correlations will occur in this model, and information loss will not be “repeatable” [12]. However, as we have discussed above, the loss of quantum coherence in processes involving a black hole may have many features which cannot be modeled by a simple system where spacetime is treated classically, so that coherence must be lost by being transferred to some internal environment rather than by falling into a spacetime singularity. Our purpose in this Appendix was merely to demonstrate that it is not necessary to violate causality or energy and momentum conservation in order to have a loss of quantum coherence.

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